Logistical Differential Equation Worksheet
AP Calculus BC

Work the following on **notebook paper**. Use your calculator on 2(b) and (c), 3(c) and (d), 4(b) and (c), and 5(c) and (d) only.

1. Suppose you are in charge of stocking a fish pond with fish for which the rate of population growth is modeled by the differential equation $\frac{dP}{dt} = 8P - 0.02P^2$.

(a) Given $P(0) = 50$.

(i) Find $\lim_{t\to\infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) Find $\frac{d^2P}{dt^2}$ and use it to find the values of P for which the solution curve is concave up and concave down. Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(b) Given $P(0) = 300$.

(i) Find $\lim_{t\to\infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

(c) Given $P(0) = 500$.

(i) Find $\lim_{t\to\infty} P(t)$.

(ii) What is the range of the solution curve?

(iii) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(iv) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(v) Does the solution curve have an inflection point? Justify your answer.

(vi) Use the information you found to sketch the graph of $P(t)$.

2. A population of animals is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = 0.01P(100 - P)$, where t is measured in years.

(a) If $P(0) = 20$, solve for P as a function of t.

(b) Use your answer to (a) and your graphing calculator to find P when $t = 3$ years.

(c) Use your answer to (a) and your graphing calculator to find t when $P = 80$ animals.

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3. A certain national park is known to be capable of supporting no more than 100 grizzly bears. Ten bears are in the park at present. The population growth of bears can be modeled by the logistic differential equation $\frac{dP}{dt} = 0.1P - 0.001P^2$, where t is measured in years.

(a) Solve for P as a function of t.

(b) Use your solution to (a) and your graphing calculator to find the number of bears in the park when $t = 3$ years.

(c) Use your solution to (a) and your graphing calculator to find how many years it will take for the bear population to reach 50 bears.

4. Suppose a virus is spreading at a local hospital holding 200 patients. The virus is spreading at a rate that is directly proportional to both the number of patients who have contracted the virus and the number of patients who have not contracted the virus. Let P be the number of patients who have contracted the virus, and let t be the time in minutes since the virus began to spread.

(a) Write a differential equation to model this rate of change.

(b) If $P(0) = 10$ and $P(15) = 50$, solve for P as a function of t.

(c) Use your solution to (b) to find the number of patients who have contracted the virus after 1 hour.

(d) Use your solution to (b) to find the time it takes for 175 patients to contract the virus.